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PERFORMANCE OF MULTIPLE TUNED MASS DAMPERS FOR TORSIONALLY COUPLED SYSTEM

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SUMMARY

Dynamic response behaviour of a simple torsionally coupled system with Multiple-Tuned Mass dampers (MTMDs) is investigated. The system is subjected to lateral excitation that is modelled as a broad-band stationary random process. MTMDs with uniformly distributed frequencies are considered for this purpose and they are arranged in a row covering the width of the system. A parametric study is conducted to investigate the effectiveness of MTMDs on reducing the response of torsionally coupled system. The parameters include the eccentricity of the main system, its uncoupled torsional to lateral frequency ratio and the damping of MTMDs. It is shown that the effectiveness of MTMDs in controlling the lateral response of the torsionally coupled system decreases with the increase in the degree of asymmetry. Further, the effectiveness of MTMDs, designed for an asymmetric system by ignoring the effect of the torsional coupling, is overestimated.

KEY WORDS: vibration control; MTMDs; torsional coupling; frequency bandwidth

INTRODUCTION

In vibration control of structures, the Tuned Mass Damper (TMD) has been accepted as an effective passive control device to suppress the structural vibration.[1,2] The TMD consists of a mass, a spring and a viscous damper attached to a vibrating main system. The natural frequency of the damper is tuned to a frequency near to the natural frequency of the main system. The vibration of the main system causes the TMD to vibrate in resonance; as a result the vibration energy is dissipated through the damping of the TMD. The determination of optimum parameters (i.e., the tuning frequency and the damping) and the effectiveness of a TMD to control structural oscillations caused by different types of excitations is now well established.[3–7]

One of the disadvantages of a single TMD is its sensitivity to the error in the computation of the natural frequency of the structure and/or that in the damping ratio of the tuned mass damper. The effectiveness of a TMD is decreased significantly by the mis-tuning or the off-optimum damping in TMD. As a result, the use of more than one tuned mass damper with different dynamic characteristics has been proposed in order to improve the effectiveness. It was shown by Iwanami and Seto[8] that two-tuned mass dampers are more effective than a single-tuned mass damper. However, the effectiveness was not significantly improved. Recently, Multiple-Tuned-Mass Dampers (MTMDs) with distributed natural frequencies were proposed by Xu and Igusa[9,10] and also studied by Yamaguchi and Harnpornchai,[11] Abe and Fujino,[12] Jangid[13] and Abe and Igusa.[14] It was shown that the MTMDs have advantages over the usual single TMD. Also, there exists an optimum frequency bandwidth for the MTMDs for which effectiveness of MTMDs is maximum.

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The review shows that considerable work is carried out on the use of TMD and MTMDs in reducing effectively the seismic response of buildings idealized as a planar model. No work is reported on the investigation of the effectiveness of MTMDs in controlling the response of a torsionally coupled system. The present study specifically addresses this problem. The objectives of the study are (i) to distinguish the difference between the dynamic behaviours of torsionally coupled (asymmetric) and uncoupled systems (symmetric) with MTMDs, (ii) to investigate how the optimum frequency bandwidth for translational and torsional responses of a torsionally coupled system varies, and (iii) to study the variation of the optimum frequency bandwidth against the important parameters such as the eccentricity of the main system, the uncoupled torsional to lateral frequency ratio and the damping of MTMDs.

SYSTEM MODEL

The system configuration consists of a main system on which n number of tuned mass dampers with different dynamic characteristics are mounted as shown in Figure 1. The main system is a torsionally coupled system, i.e. the Centre of Resistance (CR) of the main system does not coincide with the Centre of Mass (CM). As a result, the main system displays torsional effects when excited in the lateral direction. The MTMDs with uniformly distributed frequencies are evenly placed about the CM of the main system in a row covering the width, b. Note that the distribution of frequencies of MTMDs can be made either by varying the stiffness or the mass of each TMD. However, manufacturing of TMDs with uniform stiffness and damping is simpler than varying the stiffness and damping properties. Therefore, the stiffness and damping ratio of each TMD is kept constant. As a result, the CR of MTMDs coincides with the CM of the main system. Further, MTMDs with identical dynamic characteristics are equivalent to a single TMD. The damping ratio and the natural frequency of the equivalent single TMD are same as those of the individual TMDs of the MTMD system. However, the mass is the sum of the masses of the TMDs. The system is subjected to a lateral force at the CM of the main system. As a result, the main system and each TMD vibrate in the lateral direction. Also, due to torsional coupling the main system undergoes torsional vibration. Thus, the total degrees-of-freedom of the combined system is $n + 2$.

Two uncoupled frequency parameters of the main system are defined as

$$\omega_s = \sqrt{\frac{k_s}{m_s}} \tag{1a}$$

$$\omega_\theta = \sqrt{\frac{k_\theta}{m_s r_s^2}} \tag{1b}$$

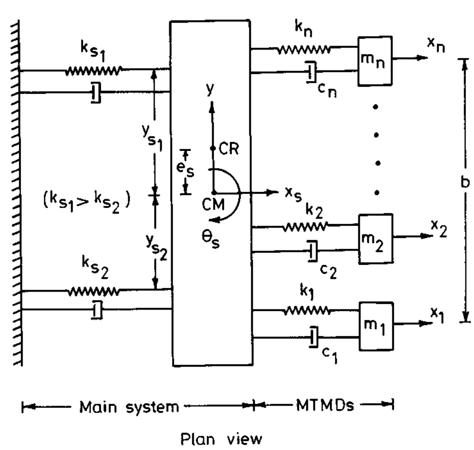

Figure 1. Model of a torsionally coupled system with multiple tuned mass dampers

where m_s is the mass of the main system, $k_s(= k_{s_1} + k_{s_2})$ is the lateral stiffness of the main system, $k_\theta(= k_{s_1} y_{s_1}^2 + k_{s_2} y_{s_2}^2)$ is the torsional stiffness of the main system about the CM, and r_s is the radius of gyration of the main system about the CM; k_{s_1} and k_{s_2} are the stiffness and y_{s_1} and y_{s_2} are the distance from the CM of the resisting elements, respectively (refer Figure 1).

The eccentricity between the CR and the CM of the main system is given by

$$e_s = \frac{k_{s_1} y_{s_1} - k_{s_2} y_{s_2}}{k_s} \tag{2}$$

The frequencies ω_s and ω_θ may be interpreted as the natural frequencies of the main system if it were torsionally uncoupled, i.e. a system with $e_s = 0$; but m_s, the mass of the main system, k_s and k_θ are the same as in the coupled system. The parameters k_{s_1}, k_{s_2} and $y_{s_1}(= y_{s_2})$ are adjusted to provide the desired values of ω_s, ω_θ and e_s.

Let ω_T be the average frequency of MTMDs (i.e. $\omega_T = (1/n) \sum_j \omega_j$) and n be the total number of MTMDs; then the natural frequency of the jth TMD is expressed as

$$\omega_j = \omega_T \left[1 + \left(j - \frac{n+1}{2} \right) \frac{\beta}{n-1} \right] \tag{3}$$

where the parameter β is the non-dimensional frequency bandwidth of MTMDs defined as

$$\beta = \frac{\omega_n - \omega_1}{\omega_T} \tag{4}$$

If k_T and ξ_T are the constant stiffness and damping ratio of each TMD, respectively, then the mass and the damping constant of the jth TMD are expressed as

$$m_j = \frac{k_T}{\omega_j^2} \tag{5}$$

$$c_j = 2\xi_T m_j \omega_j \tag{6}$$

The ratio of the total mass, m_T, of MTMDs to the mass of the main system, (m_s), is defined as the mass ratio, i.e.

$$\gamma = \frac{\sum_j m_j}{m_s} = \frac{m_T}{m_s} \tag{7}$$

The constant stiffness required for each TMD can be evaluated as

$$k_T = \frac{\gamma m_s}{\sum_j 1/\omega_j^2} \tag{8}$$

Since the torsionally coupled system with one-way eccentricity is characterized by two natural frequencies, it is difficult to define a tuning frequency ratio for MTMDs as it is done for an uncoupled (single degree-of-freedom) system. Moreover, the average frequency of MTMDs corresponds only to the lateral mode of vibration. Keeping these in view, two different tuning frequency ratios are considered in the study, namely;

$$f_1 = \frac{\omega_T}{\omega_s} \quad \text{and} \quad f_2 = \frac{\omega_T}{\omega_{s_1}} \tag{9}$$

where ω_s is the uncoupled natural frequency of lateral vibration defined earlier (Equation (1a)) and ω_{s_1} is the first natural frequency of the torsionally coupled main system.

EQUATIONS OF MOTION

The $n+2$ equations of motion of the system in Figure 1 are expressed by

$$[M]\{\ddot{X}\} + [C]\{\dot{X}\} + [K]\{X\} = \{1\} f(t) \tag{10}$$

where $\{X\} = \{x_s, \theta_s, x_1, x_2, \ldots, x_n\}^T$ is the displacement vector of the system model, x_s and θ_s are the displacement and rotation of the main system, x_j ($j = 1, 2, \ldots, n$) is the displacement of the jth tuned mass damper, $f(t)$ is the lateral excitation force acting at the CM of the main system, $\{1\} = \{1, 0, 0, \ldots, 0\}^T$, $[M]$, $[C]$ and $[K]$ are the mass, damping and stiffness matrices of size $(n+2) \times (n+2)$ respectively. The matrices $[M]$, $[C]$ and $[K]$ are expressed as

$$[M] = \text{diag}\,[m_s, m_s r_s^2, m_1, m_2, \ldots, m_n] \tag{11}$$

$$[C] = \begin{bmatrix} c_s + \sum c_j & c_{s\theta} + \sum c_j y_j & -c_1 & -c_2 & \cdots & -c_n \\ & c_\theta + \sum c_j y_j^2 & -c_1 y_1 & -c_2 y_2 & \cdots & -c_n y_n \\ & & c_1 & 0 & \cdots & 0 \\ & & & c_2 & \cdots & 0 \\ & & & & \cdots & \vdots \\ \text{sym} & & & & & c_n \end{bmatrix} \tag{12}$$

$$[K] = \begin{bmatrix} k_s + \sum k_j & k_{s\theta} + \sum k_j y_j & -k_1 & -k_2 & \cdots & -k_n \\ & k_\theta + \sum k_j y_j^2 & -k_1 y_1 & -k_2 y_2 & \cdots & -k_n y_n \\ & & k_1 & 0 & \cdots & 0 \\ & & & k_2 & \cdots & 0 \\ & & & & \cdots & \vdots \\ \text{sym} & & & & & k_n \end{bmatrix} \tag{13}$$

where c_s, $c_{s\theta}$ and c_θ are the elements of the damping matrix of the main system without MTMDs which are obtained by assuming a modal damping, ξ_s, in both modes of vibration, $k_{s\theta}$ $(= k_s e_s)$ is the coupling term between the trnaslational and the torsional degrees-of-freedom for the main system, and y_j is the distance of the jth TMD from the CM of the main system.

The steady-state harmonic response of the system to harmonic excitation, $f(t) = e^{i\omega t}$ (where ω is the circular frequency and $i = \sqrt{-1}$) is given by $\{X\} = X(\omega) e^{i\omega t}$. The amplitude vector of the steady-state response, $X(\omega)$, is given by

$$X(\omega) = (-\omega^2[M] + i\omega[C] + [K])^{-1}\{1\} \tag{14}$$

The first two elements of the vector $X(\omega)$ are the amplitudes of the lateral and torsional displacements (referred as $x_s(\omega)$ and $\theta_s(\omega)$) of the main system, respectively. If the external excitation is modelled as a stationary random process characterized by its Power Spectral Density Function (PSDF), then the PSDF of the response of the main system is given by[15]

$$S_{x_s}(\omega) = |x_s(\omega)|^2\, S_f(\omega) \tag{15a}$$

$$S_{\theta_s}(\omega) = |\theta_s(\omega)|^2\, S_f(\omega) \tag{15b}$$

where $S_f(\omega)$ is the PSDF of the excitation force, $f(t)$. The mean square responses of the main system are expressed as

$$\sigma_{x_s}^2 = \int_{-\infty}^{\infty} S_{x_s}(\omega)\, d\omega \tag{16a}$$

$$\sigma_{\theta_s}^2 = \int_{-\infty}^{\infty} S_{\theta_s}(\omega)\, d\omega \tag{16a}$$

NUMERICAL STUDY

The main system is characterized by the uncoupled lateral frequency, ω_s, the normalized eccentricity ratio, e_s/r_s, the ratio of uncoupled torsional to lateral frequency, ω_θ/ω_s and the damping ratio, ξ_s. On the other hand, MTMDs are characterized by the mass ratio, γ, the total number of TMDs, n, the damping ratio, ξ_T, the normalized frequency bandwidth, β, the tuning frequency ratio, f_1 or f_2, and the normalized width of placement, b/r_s. However, to reduce the number of parameters, some of these parameters are held constant. These are: $\xi_s = 2$ per cent, $\gamma = 1$ per cent, $n = 21$ and $b/r_s = 1$. The value of $n = 21$ is based on the findings of the earlier studies where it was shown that the response of the system is not significantly influenced if the number of TMDs is increased beyond 21.[11, 13] Torsional coupling (the degree of asymmetry) of the main system depends upon the parameters e_s/r_s and ω_θ/ω_s. Here, four values of the e_s/r_s (i.e. 0, 0·1, 0·2 and 0·3) and three values of the ω_θ/ω_s (i.e. 0·5, 1 and 2) are considered. Note that $e_s/r_s = 0$ refers a symmetric system and $\omega_\theta/\omega_s = 0{\cdot}5$, 1 and 2 indicates a torsionally flexible, strong torsionally coupled and torsionally stiff main system respectively. Further, the tuning frequency ratio of MTMDs (f_1 and f_2) is taken as unity. The system is subjected to a lateral excitation and its power spectral density function is modelled as a stationary white-noise random process, i.e.

$$E[f(t)f(t+\tau)] = 2\pi S_0\, \delta(\tau) \tag{17}$$

where E is the expectation operator, S_0 is the intensity of white-noise excitation and $\delta(\cdot)$ is dirac-delta function.

The response quantities of interest are the Root Mean Square (r.m.s.) values of the lateral and torsional displacements of the main system with and without MTMDs. The responses are expressed as the response ratio R defined by

$$R = \frac{\text{r.m.s. response of the main system with MTMDs}}{\text{r.m.s. response of the main system without MTMDs}} \tag{18}$$

The ratio R is a measure of the effectiveness of MTMDs. The value less than unity implies that MTMDs are effective for vibration control. The other response quantity of interest is the optimum frequency bandwidth (β^{opt}) of MTMDs. For a given structural system and a specific excitation, the optimum frequency bandwidth is defined as that which produces the minimum response ratio R. The results are first obtained for MTMDs tuned to the uncoupled lateral frequency of the main system (i.e. $f_1 = 1$), and their damping ratio, $\xi_T = 1\%$.

In Figure 2, the variation of the response ratio R for x_s and θ_s is plotted against the frequency bandwidth, β for $\omega_\theta/\omega_s = 1$. R for both responses increases as the eccentricity increases. This shows that if MTMDs are designed for asymmetric buildings by ignoring the effect of torsional coupling, then the effectiveness of MTMDs is overestimated. The variation of the β^{opt} and the corresponding values of R with eccentricity is shown in Table I. It is seen that the β^{opt} increases with the increase in the eccentricity for both translational and torsional responses. However, the β^{opt} for the torsional response is comparatively less than that for the translational response. Note that the optimum frequency bandwidth for the translational response at $e_s/r_s = 0{\cdot}3$ would be about 3·5 times more than that for the corresponding symmetric system. Further, it is seen that the effectiveness of MTMDs in reducing both translational and rotational responses decreases as the eccentricity ratio increases. For $e_s/r_s = 0{\cdot}3$, the reductions in translational and torsional responses are about 8–9 per cent; for $e_s/r_s = 0$, the reduction in translational response is about 33 per cent.

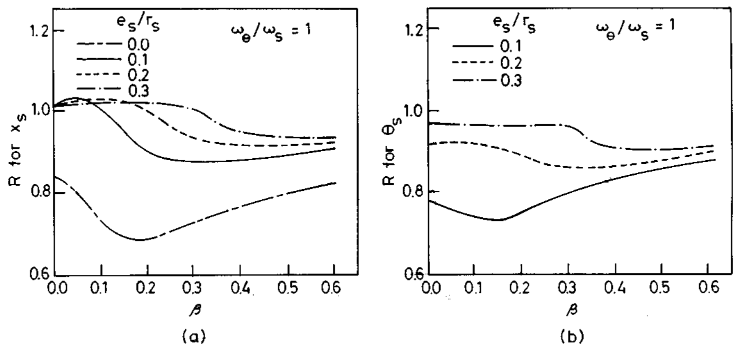

Figure 2. Variation of the response ratio with the frequency bandwidth for $\omega_\theta/\omega_s = 1$, $\xi_T = 1$ per cent and $f_1 = 1$: (a) for translation; (b) for torsion

Table I. Optimum frequency bandwidth and effectiveness of the MTMDs ($\omega_\theta/\omega_s = 1$, $\xi_T = 1\%$ and $f_1 = 1$)

	x_s		θ_s	
e_s/r_s	β^{opt}	R	β^{opt}	R
0	0·17	0·6743	—	—
0·1	0·30	0·8718	0·15	0·7286
0·2	0·44	0·9123	0·28	0·8640
0·3	0·60	0·9292	0·40	0·9110

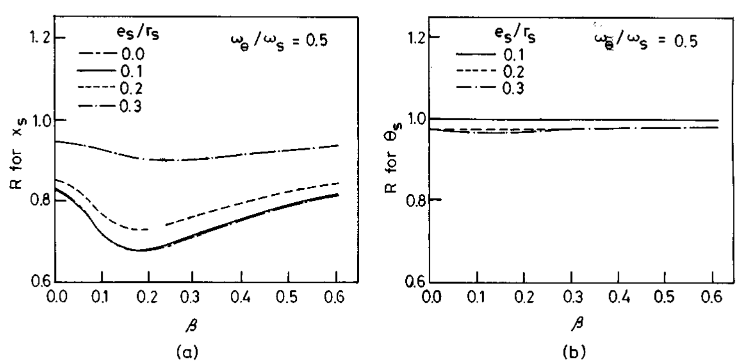

Figure 3. Variation of the response ratio with the frequency bandwidth for $\omega_\theta/\omega_s = 0{\cdot}5$, $\xi_T = 1$ per cent and $f_1 = 1$: (a) for translation; (b) for torsion

Figure 3 shows the same variations for torsionally coupled system with $\omega_\theta/\omega_s = 0{\cdot}5$. For small eccentricity ratio ($e_s/r_s = 0{\cdot}1$), it is seen that the variation of R (for translational response) with the frequency bandwidth is nearly the same as that for $e_s/r_s = 0$. This is due to the fact that translational response for $\omega_\theta/\omega_s = 0{\cdot}5$ is predominently governed by the second mode of vibration which has insignificant torsion for small eccentricity. However, as e_s/r_s increases, this variation significantly differs from that for $e_s/r_s = 0$ because of greater torsional coupling. Further, the optimum frequency bandwidth does not significantly vary with e_s/r_s for $\omega_\theta/\omega_s = 0{\cdot}5$ (see also Table II)

Table II shows the reduction of both translational and torsional responses at β^{opt} for $\omega_\theta/\omega_s = 0{\cdot}5$. It is seen that the reduction in torsional response is small. This is due to the fact that for a given value of e_s/r_s ($\neq 0$), the

Table II. Optimum frequency bandwidth and effectiveness of the MTMDs ($\omega_\theta/\omega_s = 0{\cdot}5$, $\xi_T = 1\%$ and $f_1 = 1$)

	x_s		θ_s	
e_s/r_s	β^{opt}	R	β^{opt}	R
0	0·17	0·6743	—	—
0·1	0·17	0·6806	0·15	0·98090
0·2	0·18	0·7326	0·16	0·98612
0·3	0·20	0·9016	0·22	0·99970

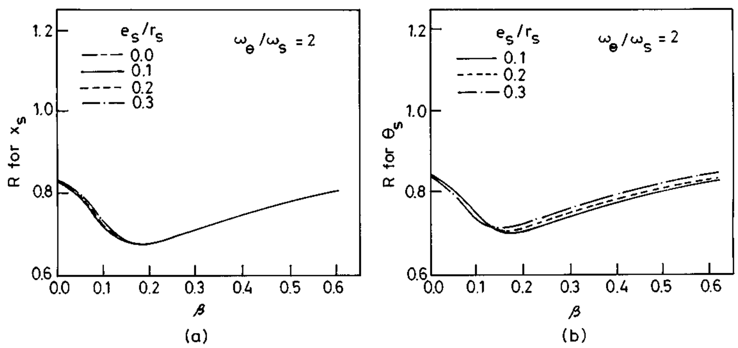

Figure 4. Variation of the response ratio with the frequency bandwidth for $\omega_\theta/\omega_s = 2$, $\xi_T = 1$ per cent and $f_1 = 1$: (a) for translation; (b) for torsion

contribution of the second mode (which has frequency close to ω_s and has significant translational component) to the overall torsional response is not large. Since MTMDs are tuned to ω_s, the torsional response contributed by the second mode is effectively controlled, but there is no substantial reduction in the overall torsional response because the second mode contribution to the overall torsional response is not significant, as stated before. Further, comparison of Tables I and II shows that the reduction in translational response due to MTMDs is more for $\omega_\theta/\omega_s = 0{\cdot}5$ than for $\omega_\theta/\omega_s = 1$. This is so because for a given value of e_s/r_s, torsional coupling is more for the latter and, therefore, the effectiveness of MTMDs becomes less for $\omega_\theta/\omega_s = 1$.

Figure 4 shows the same variations for $\omega_\theta/\omega_s = 2$, i.e. torsionally stiff system with lateral and torsional frequencies well separated. The variation of R for the translational and the torsional responses with the frequency bandwidth are almost the same for all values of e_s/r_s (including $e_s/r_s = 0$). Thus, the effectiveness of MTMDs and the value of β^{opt} remain nearly the same for all values of e_s/r_s. Further, Table III shows that the reduction in the torsional response is of the same order as that for the translational response and is about 30 per cent. This happens due to the fact that both responses are primarily governed by the first mode of vibration with frequency close to ω_s and, therefore, they are effectively controlled by MTMDs for all values of e_s/r_s.

The above results are obtained for MTMDs with $\xi_T = 1$ per cent. The same parametric study is conducted for $\xi_T = 5$ percent and the results are shown in Figures 5–7. It is seen from the figures that the nature of variation of R with β is similar to that obtained for $\xi_T = 1$ per cent. However, the values of β^{opt} and the corresponding values of R are different than those for $\xi_T = 1$ per cent. For higher damping of MTMDs, the values of β^{opt} decrease, while the corresponding values of R increase (compare between Figures 2–4 and 5–7).

It is seen from Figures 3–7 that MTMDs are more effective in controlling the response than single TMD. Note that the values of R for single TMD correspond to those for $\beta = 0$. Table IV shows the ratio $R(\beta^{opt})/(R(\beta = 0)$ for different combinations of ω_θ/ω_s and e_s/r_s. It is seen that this ratio for the translational

Table III. Optimum frequency bandwidth and effectiveness of the MTMDs ($\omega_\theta/\omega_s = 2$, $\xi_T = 1\%$ and $f_1 = 1$)

	x_s		θ_s	
e_s/r_s	β^{opt}	R	β^{opt}	R
0	0·17	0·67430	—	—
0·1	0·18	0·67417	0·15	0·69732
0·2	0·18	0·67419	0·15	0·71159
0·3	0·18	0·67588	0·16	0·71655

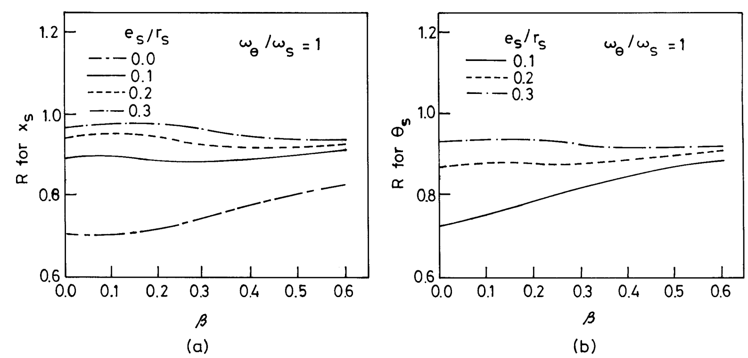

Figure 5. Variation of the response ratio with the frequency bandwidth for $\omega_\theta/\omega_s = 1$, $\xi_T = 5$ per cent and $f_1 = 1$: (a) for translation; (b) for torsion

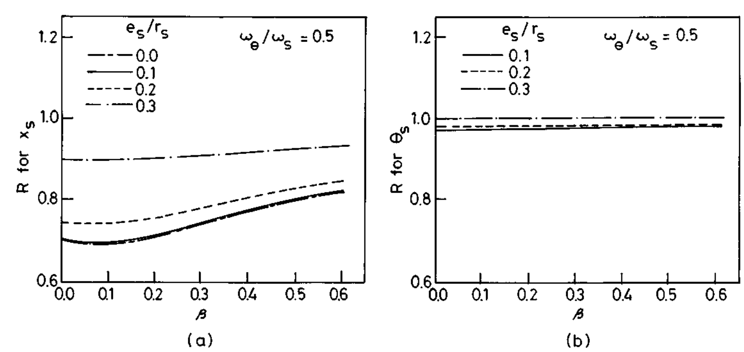

Figure 6. Variation of the response ratio with the frequency bandwidth for $\omega_\theta/\omega_s = 0·5$, $\xi_T = 5$ per cent and $f_1 = 1$: (a) for translation; (b) for torsion

response increases with increase in e_s/r_s (for $\omega_\theta/\omega_s = 0·5$ and 1), i.e. the relative effectiveness of MTMDs in comparison to single TMD decreases as e_s/r_s increases. However, for the torsionally stiff system ($\omega_\theta/\omega_s = 2$), the relative advantage of MTMDs (in comparison to single TMD) does not practically change with the change in e_s/r_s (for the translational response).

The foregoing discussions clearly demonstrate that the effectiveness of MTMDs in reducing the lateral response of asymmetric system is less than that for the corresponding symmetric system. For strong torsionally coupled system ($\omega_\theta/\omega_s = 1$), benefit of MTMDs for lateral response becomes marginal for $e_s/r_s = 0·3$. However, as the asymmetric system becomes torsionally more stiff (i.e. $\omega_\theta/\omega_s > 1$), the control of

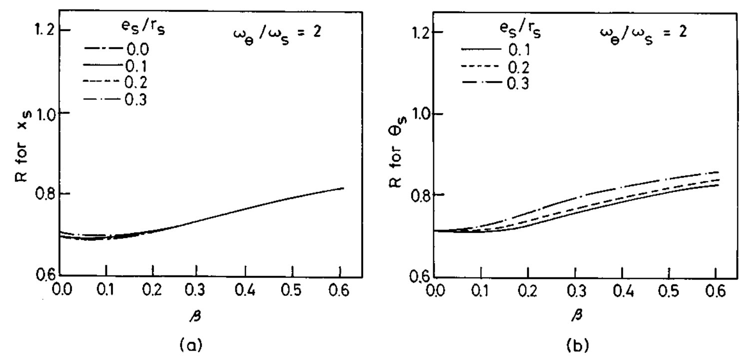

Figure 7. Variation of the response ratio with the frequency bandwidth for $\omega_\theta/\omega_s = 2$, $\xi_T = 5$ per cent and $f_1 = 1$: (a) for translation; (b) for torsion

Table IV. Relative effectiveness of the MTMDs with respect to a single TMD ($\xi_T = 1\%$ and $f_1 = 1$)

	$R(\beta^{opt})/R(\beta=0)$					
	$\omega_\theta/\omega_s = 0{\cdot}5$		$\omega_\theta/\omega_s = 1$		$\omega_\theta/\omega_s = 2$	
e_s/r_s	x_s	θ_s	x_s	θ_s	x_s	θ_s
0	0·8106	—	0·8106	—	0·8106	—
0·1	0·8180	0·9989	0·8622	0·9384	0·8108	0·7589
0·2	0·8582	0·9992	0·9123	0·9387	0·8109	0·8341
0·3	0·9507	1.0	0·9292	0·9389	0·8109	0·8360

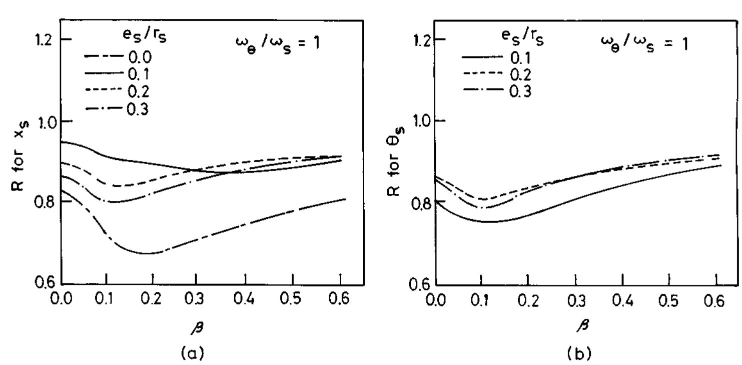

Figure 8. Variation of the response ratio with the frequency bandwidth for $\omega_\theta/\omega_s = 1$, $\xi_T = 1$ per cent and $f_2 = 1$: (a) for translation; (b) for torsion

the lateral response improves. For $\omega_\theta/\omega_s = 2$, MTMDs provide about 30 per cent reduction in response for $e_s/r_s = 0{\cdot}3$. Note that there is also a substantial reduction in torsional response of the system because of torsional coupling. The above observations are made for a tuning frequency ratio, $f_1 = 1$. In order to investigate whether there exists a better tuning frequency for which the reduction in response for strong torsionally coupled system becomes adequate for $e_s/r_s = 0{\cdot}3$, a tuning frequency ratio of $f_2 = 1$ (refer equation (9)) is tried.

The results are shown in Figure 8 and Table V. It is seen that the reduction in responses (for both translation and torsion) is about 20 per cent for $e_s/r_s = 0{\cdot}3$. The effectiveness of MTMDs in reducing the

Table V. Optimum frequency bandwidth and effectiveness of the MTMDs ($\omega_\theta/\omega_s = 1$, $\xi_T = 1\%$ and $f_2 = 1$)

	x_s		θ_s	
e_s/r_s	β^{opt}	R	β^{opt}	R
0	0·17	0·6743	—	—
0·1	0·33	0·8770	0·10	0·7473
0·2	0·13	0·84505	0·11	0·80245
0.3	0·13	0·80397	0·12	0·78596

response for $e_s/r_s = 0{\cdot}3$ and 0·2 improve (cf. Tables I and V) when the tuning frequency ratio is taken as f_2 instead of f_1. However, for $e_s/r_s = 0{\cdot}1$ the relative advantage of using this tuning frequency ratio is marginal. Note that when the tuning frequency ratio f_2 is used, the reduction in the translational response is more for $e_s/r_s = 0{\cdot}3$ than for $e_s/r_s = 0{\cdot}2$ and 0·1. This is opposite to the trend observed for the case of the tuning frequency ratio, f_1. The reason for this may be attributed to the spacing of the natural frequencies. For smaller e_s/r_s (i.e. for 0·1), natural frequencies are closely spaced and, hence, the contribution of the mode other than the one which is being suppressed, significantly influences the response. As a result, the reduction in the total response is not substantial.

From the limited numerical study, it appears that there exists no unique value for the tuning frequency for achieving the maximum reduction of response for asymmetric systems. It primarily depends upon the the parameters ω_θ/ω_s and e_s/r_s. However, it is possible to obtain a tuning frequency for a given combination of ω_θ/ω_s and e_s/r_s, for which the response reduction would be maximum. This could be achieved by trial and error or by running an optimization program. In this way, the effective design of MTMDs can be realized for a meaningful reduction of the response of asymmetric systems having realistic range of the degree of asymmetry.

CONCLUSION

Performance of MTMDs for controlling the response of a torsionally coupled system is investigated. A simple one-way eccentric model having two degrees of freedom is provided with MTMDs. The system is subjected to lateral excitation that is modelled as a white-noise stationary random process. The reduction in response of the torsionally coupled system due to MTMDs is investigated for a number of parametric variations. The results of the study lead to the following conclusions:

1. Effectiveness of MTMDs in controlling the translational response is less for an asymmetric system than the corresponding symmetric system. Therefore, if MTMDs are designed for asymmetric buildings by ignoring the effects of their torsional coupling, then the effectiveness of MTMDs is overestimated.
2. However, for torsionally very stiff asymmetric buildings ($\omega_\theta/\omega_s \geqslant 2$), the design of MTMDs by ignoring the effects of torsional coupling is justified.
3. Optimum frequency bandwidth of MTMDs changes with the change in the eccentricity of the asymmetric system. Therefore, optimum frequency bandwidth of MTMDs computed by ignoring the effects of torsional coupling may not effectively control the response of the asymmetric building.
4. For strong torsionally coupled system ($\omega_\theta/\omega_s = 1$), the reduction in response for the torsion is more than that for the translation. Moreover, the optimum frequency bandwidth required for controlling effectively the torsional response is less than that required for the translational response.
5. The increase in the damping of MTMDs decreases the optimum frequency bandwidth, but it also decreases the effectiveness of MTMDs in controlling the response.
6. MTMDs are more effective than single TMD even for torsionally coupled system. However, the relative advantage of MTMDs (compared to single TMD) decreases with increase in the eccentricity ratio.

7. Tuning frequency of MTMDs for which the maximum response reduction is achieved for asymmetric system depends upon the degree of asymmetry (ω_θ/ω_s and e_s/r_s).
8. For strong torsionally coupled system ($\omega_\theta/\omega_s = 1$), better reduction in the translational response for higher eccentricity (i.e. $e_s/r_s = 0{\cdot}3$) can be achieved for a tuning frequency equal to the first natural frequency of the coupled system. For torsionally stiffer system ($\omega_\theta/\omega_s > 1$), a tuning frequency equal to the uncoupled lateral frequency provides better reduction in response

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